8 Critical Currents

In view of technical applications it is of interest that superconductors remain in the superconducting state even under extreme experimental conditions. Therefore, one aims at first for large critical fields B_{c2}. As the thermodynamic critical field B_c^{th} is usually quite small, we read off from (6.36) that one needs materials with a large Ginzburg-Landau parameter $\varkappa$. But in addition, we have also to take into account that superconductivity can break down for a large enough current. Therefore, another aim is to achieve a large critical current. The latter point has not yet been mentioned and has to be discussed separately for type I and type II superconductors, respectively.

In superconductors of type I no flux lines exist. Therefore it is possible to connect directly the critical current with the critical field.

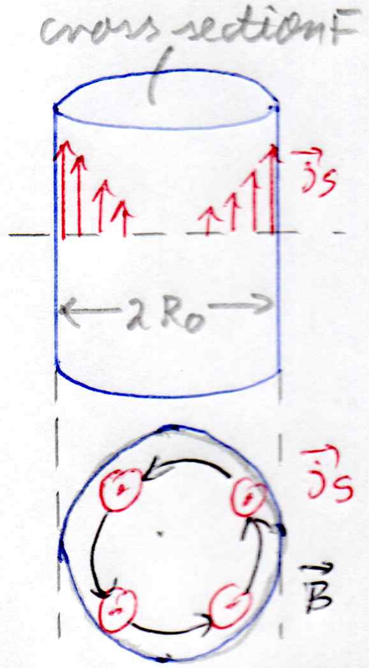
In contrast to that the physical properties in the Shubnikov phase of type II superconductors are governed by the flux lines. A current of superconducting electrons at a non-vanishing magnetic field is affected by Lorentz forces and, conversely, this leads also to forces upon the flux lines. But a motion of the flux lines would lead due to several reasons to energy dissipation and, thus, to a breakdown of superconductivity. Therefore, one needs pinning forces at lattice defects, which prevent such a motion of flux lines. On the other hand, the presence of lattice defects can have the unwanted consequence that the critical temperature of superconductivity is reduced.

8.1 Critical Current at Type I Superconductors:

A current of superconducting electrons generates according to the Oersted law a magnetic field. This leads to the question how large the current of superconducting electrons can be maximally be until the superconducting phase breaks down as the critical field is reached. Thus, we are interested in the relation between the critical current I_c and the critical magnetic field H_c:

$$I_c = I_c(H_c) \tag{8.1}$$

8.1.1 Calculation:



In order to determine (8.1) we consider the set-up of a cylindrical conductor through which a superconducting current flows. The superconducting current density $\vec{j}_s(\vec{r})$ through the cross section area F leads to the current

$$I = \int_F \vec{j}_s(\vec{r}) \cdot d\vec{F} \tag{8.2}$$

Furthermore, we have to take into account the Oersted law

$$\mathrm{rot}\,\vec{B}(\vec{r}) = \mu_0 \vec{j}_s(\vec{r}) \tag{8.3}$$

Integrating (8.3) over the cross section F yields together with the Stokes theorem

$$\int_F \mathrm{rot}\,\vec{B}(\vec{r}) \cdot d\vec{F} = \oint_{\partial F} \vec{B}(\vec{r}) \cdot d\vec{s} = \mu_0 \int_F \vec{j}(\vec{r}) \cdot d\vec{F} \overset{(8.2)}{=\!=} \mu_0 I \tag{8.4}$$

Due to the cylinder symmetry of the above set-up, the magnetic induction $\vec{B}$ is constant along the boundary ∂F, yielding

$$I(B) = 2\pi R_0 \frac{B}{\mu_0} \Rightarrow I(H) = 2\pi R_0 H \tag{8.5}$$

Thus, the resulting current I is proportional to the magnetic field strength H. The critical current I_c then follows from (8.5) for the case that the magnetic field strength reaches the critical value H_c:

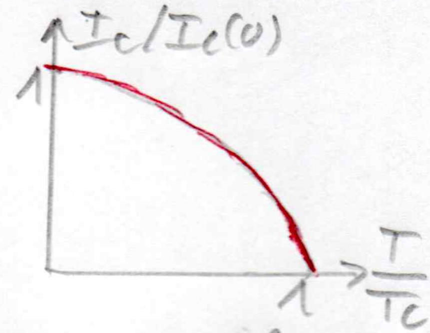
$$I_c = I(H_c) \overset{(8.5)}{=\!=} 2\pi R_0 H_c \tag{8.6}$$

8.1.2 Example:

Let us consider tin (Sn) as an example for a type I superconductor. As its critical induction amounts to $B_c = 0.08\,T$, we obtain for a cylinder of radius $R_0 = 1\,mm$ from (8.6) the critical current $I_c = 400\,A$.

8.1.3 Temperature dependence:

The critical field H_c is temperature dependent, which is described by the rough approximation (1.3). Therefore, (8.6) implies that then this temperature dependence is inherited by the critical current



$$I_c(T) = I_c(0)\left\{1 - \frac{T^2}{T_c^2}\right\} \qquad (8.7)$$

8.1.4 Current Density:

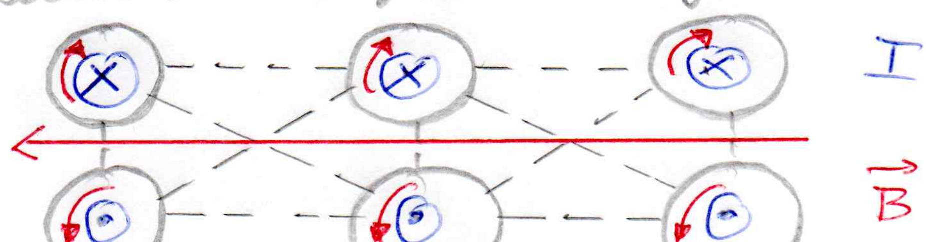
According to the London theory the current of superconducting electrons flows only within a layer, whose thickness is given by the London penetration length λ_L. In view of an estimate we assume that the superconducting current density is is constant within the whole layer of thickness λ_L and obtain from the critical current I_c for the critical current density

$$j_{sc} = \frac{I_c}{F} = \frac{I_c}{2\pi R_0 \lambda_L} \qquad (8.8)$$

In the above cylinder of tin (Sn) we have $\lambda_L = 4 \cdot 10^{-8}$ m, so get from (8.8) the gigantic critical current density $j_{sc} = 1.6 \cdot 10^8$ A/cm^2.

8.1.5 Coils:

For coils one has to take into account that the magnetic field at a single winding is enhanced by the magnetic field of the neighbouring windings:



For the construction of a superconducting coil one has, therefore, to take into account the fact that the critical current at a winding is smaller than the corresponding critical current at a single winding:

$$I_c(\text{coil}) < I_c(\text{single winding}) \qquad (8.9)$$

8.2 Critical Current at Type II Superconductors:

For type II superconductors we have to discuss the critical currents separately in the Meissner and in the Shubnikov phase.

8.2.1 Lower Critical Field:

From the discussion of the critical fields in Chapter 6 we conclude that for type II superconductors the geometric mean of the lower and the upper critical field B_{c1} and B_{c2} is approximately given by the thermodynamic critical field B_c^{th}:

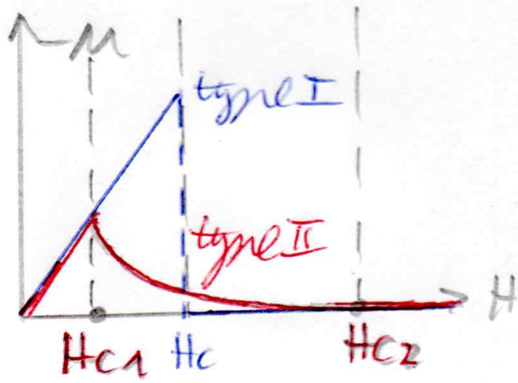
$$\sqrt{B_{c1}\cdot B_{c2}} \overset{(6.36),(6.75)}{=} \sqrt{\ln\varkappa+\frac{7}{12}}\cdot B_c^{th} \approx B_c^{th} \qquad (8.10)$$

Here we have neglected the dimensionless prefactor $\sqrt{\ln\varkappa + 7/12}$ as it is usually of the order of one. Furthermore, we know that type I and type II superconductors have approximately the same thermodynamic critical field, so the critical field B_c of a type I superconductor coincides roughly with the thermodynamic critical field of a type II superconductor:

$$B_c \approx B_c^{th} \qquad (8.11)$$

Thus from (8.10) and (8.11) we conclude

$$B_c \approx \sqrt{B_{c1}\cdot B_{c2}} \qquad (8.12)$$



The approximate validity of (8.12) can be checked experimentally for the transition from the type I superconductor lead (Pb) to a type II superconductor via successive doping with indium (In).

Furthermore, we read off from (8.12) that a type II superconductor has a lower critical field B_{c1}, which is typically smaller than the critical field B_c of a type I superconductor:

$$B_{c1} < B_c \qquad (8.13)$$

8.2.2 Lower Critical Current:

During the Meissner phase of a type II superconductor no flux quanta exist. Therefore the reasoning of subsection 8.1.1 can be taken over for obtaining the lower critical current:

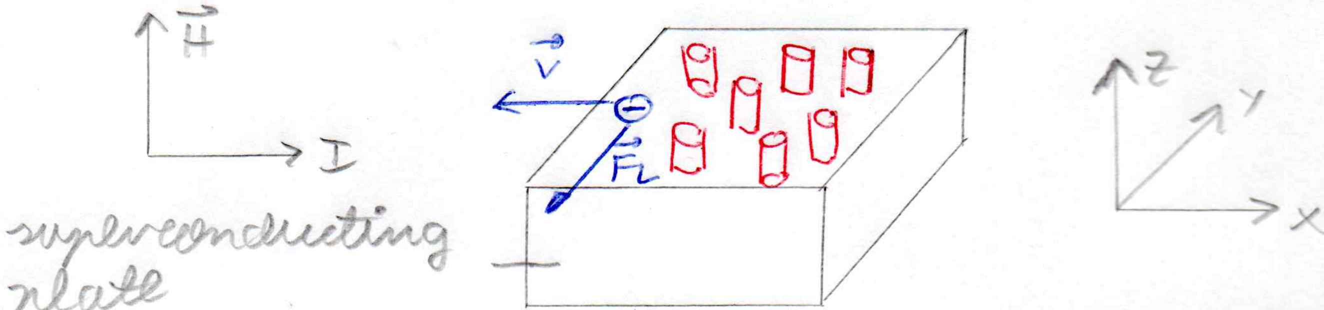
$$I_{c1} = 2\pi R_0 H_{c1} \qquad (8.14)$$

Then we conclude from (8.13) that the lower critical current of a type II superconductor is smaller than the critical current of a type I superconductor

$$I_{c1} < I_c \tag{8.15}$$

8.2.3 Geometry:

The question arises now how the presence of flux lines in the Shubnikov phase affect the critical current. To this end we consider the following geometry:



The flux lines penetrate the plate from the bottom to the top. Here it is essential that the external magnetic field goes basically unaffected through the core of the flux lines.

8.2.4 Pinning Force:

Let us consider a current of electrons moving horizontally through the superconducting plate. When an electron moves through a flux line, the magnetic induction penetrating the core of the flux line creates a Lorentz force, which deflects the electron:

$$\vec{F}_L = e\,\vec{v} \times \vec{B} \tag{8.16}$$

But due to the Newton axiom "actio = -reactio" also the electron is pressing against the flux line with the force

$$\vec{F}_{\phi_0} = -\vec{F}_L \tag{8.17}$$

As there are many electrons in the superconductor, we have to sum up over all the respective forces (8.17), which they exert

$$\vec{F}_P = -\sum_i \vec{F}_{Li} \underset{=}{\overset{(8.16)}{}} -\sum_i \vec{v}_i \times \vec{B}(\vec{r}_i) \tag{8.18}$$

Going over from a discrete sum to a continuous integral necessitates to introduce the electron density

$$\sum_i \,\bullet = \int_V dV\; n_s(\vec{r})\,\bullet \tag{8.19}$$

so (8.18) goes over into

$$\vec{F}_P = -e \int_V dV \, n_s(\vec{r}) \, \vec{v} \times \vec{B}(\vec{r}) \tag{8.20}$$

Due to the cylinder symmetry of the geometry we can decompose the volume integral like in (6.48)

$$\vec{F}_P = -e \int_F n_s(\vec{r}) \, \vec{v} \times \vec{B}(\vec{r}) \cdot dF \, L \tag{8.21}$$

Here L denotes the extension of the plate in z-direction. Assuming that the superconducting electron current density

$$\vec{j}_s(\vec{r}) = -e \, n_s(\vec{r}) \, \vec{v} \tag{8.22}$$

is spatially constant all over the plate, (8.21) reduces to

$$\vec{F}_P = \vec{j}_s \times \int_F \vec{B}(\vec{r}) \cdot dF \, L \tag{8.23}$$

As we have $\vec{j}_s = j_s \vec{e}_x$ and $\vec{B}(\vec{r}) = B(\vec{r}) \vec{e}_z$, we get $\vec{e}_x \times \vec{e}_z = -\vec{e}_y$ and the area integral yields a multiple of the elementary flux quantum

$$\int_F B(\vec{r}) \, dF = n \phi_0 \tag{8.24}$$

As a result we obtain

$$\vec{F}_P = - j_s \, n \phi_0 \, L \, \vec{e}_y \tag{8.25}$$

In order to prevent that the flux lines move under the impact of this force, they must be anchored with a pinning force.

8.2.5 Interaction Mechanism:

How can one imagine microscopically the interaction between the moving electrons and the flux lines? The force emerges as the current transporting electrons interact with the superconducting electrons. The latter encircle the flux quanta in order to prevent the magnetic induction to enter the volume of the superconductor.

8.2.6 Spatial Averaging:

Now we go to a mesoscopic length scale and consider the discrete flux lines with a continuum description. To this end we denote with $\bar{B}$ the spatially averaged magnetic induction per unit cell in the flux line lattice. Furthermore, F_{ϕ_0} stands for the area of a unit cell in the flux line lattice. Then we have

$$\tilde{B} = \frac{\phi_0}{F\phi_0} \tag{8.26}$$

so we conclude from (8.25)

$$\frac{\vec{F}_P}{V} = \vec{j}_s \times \vec{\tilde{B}} \tag{8.27}$$

with the volume $V = n F\phi_0 L$. Inserting in (8.27) the Oersted law (8.3) yields

$$\frac{\vec{F}_P}{V} = \frac{1}{\mu_0} \operatorname{rot} \vec{\tilde{B}} \times \vec{\tilde{B}} \tag{8.28}$$

Thus, we obtain with

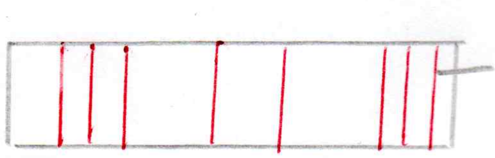
$$\vec{\tilde{B}} = \tilde{B}\vec{e}_z, \quad \operatorname{rot}\vec{\tilde{B}} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & \tilde{B} \end{vmatrix} = \partial_y \tilde{B}\vec{e}_x \overset{(8.3)}{=\!=} \mu_0 \vec{j}_s \tag{8.29}$$

for the pinning force per volume

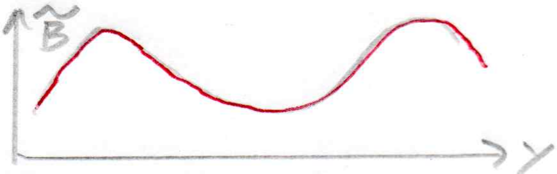
$$\frac{\vec{F}_P}{V} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ j_s & 0 & 0 \\ 0 & 0 & \tilde{B} \end{vmatrix} = -j_s \tilde{B}\vec{e}_y \overset{(8.29)}{=\!=} -\frac{1}{\mu_0}\tilde{B}\frac{\partial\tilde{B}}{\partial y}\vec{e}_y \tag{8.30}$$

Thus, the pinning force per volume is connected with a current or a gradient of the magnetic field. Therefore, the Shubnikov phase is characterised by an inhomogeneous distribution of the magnetic induction:

cut through a superconducting layer



spatial variation of averaged magnetic induction



Here a large number of discrete flux lines corresponds on average to a large magnetic induction.

8.3 Loss Mechanisms:

Are the pinning forces too small, then the flux lines move back and forth in an external magnetic field. There are two loss mechanisms, which we discuss in due course, leading to a local heating and, consequently, to a breakdown of superconductivity.

In order to avoid an overheating within the superconductor, an Ohmic resistance can be added at the input leads to the superconductor. Then the Ohmic heat is produced out of the superconductor and not inside.

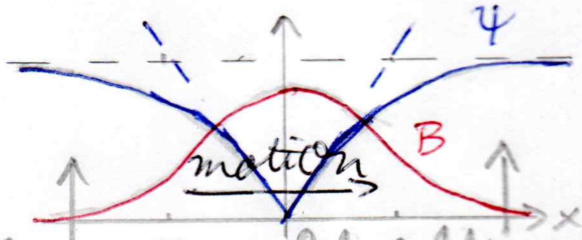
A magnetizable object should never be brought nearby a superconductor. High magnetic fields would attract the object. But then, due to the Newton "actio = − reactio", large forces would also act on the flux lines, so they could be torn from their respective pinning centers. This would lead to a sudden breakdown of superconductivity.

8.3.1 First Loss Mechanism:

Moving flux quanta generate a time dependent magnetic induction. Due to the induction law $\mathrm{rot}\,\vec{E} = -\dot{\vec{B}}$ this leads to induction voltages, which switch on induced eddy currents ("Wirbelströme") in the superconductor. But these eddy currents of normal conducting electrons yield a release of Joule heat.

8.3.2 Second Loss Mechanism:

The center of a flux quantum is normal conducting, thus it does not contain Cooper pairs. Therefore a motion of a flux quantum leads to two processes:



creation of Cooper pairs breakdown of Cooper pairs

- In front of the flux quantum Cooper pairs have to be broken, which costs energy.
- Behind the flux quantum Cooper pairs have to be created, which releases energy.

The motion of a flux quantum can be considered to be slow provided that the time scale for the motion τ_1 is large in comparison with the time scale τ_2 to break and create Cooper pairs. Thus, such an adiabatic motion is characterized by $\tau_1 \gg \tau_2$. Then there is enough time for a local exchange of energy. Locally, breaking the Cooper pairs costs the same amount of energy as the subsequent creation of Cooper pairs.

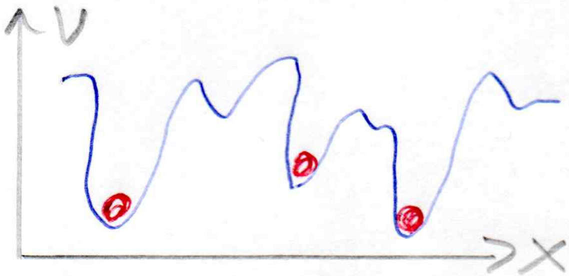
But in the opposite case $\tau_1 \ll \tau_2$ of a fast motion of a flux quantum the breaking of Cooper pairs occurs at a larger magnetic field, which costs more energy.

In contrast to that the recombination of electrons to Cooper pairs occurs at a smaller magnetic field, so the energy gain is then smaller. Therefore, in total, a fast motion of flux quanta is connected with a dissipation of energy.

8.4 Anchoring of Flux Quanta by Lattice Inhomogeneities

Quantitatively the anchoring of flux quanta by lattice inhomogeneities can be modelled phenomenologically within the realm of the Ginzburg-Landau theory by introducing spatially dependent Landau parameters $\alpha(\vec{r})$ and $\beta(\vec{r})$. In view of treating such a spatial dependence perturbatively, we make the ansatz

$$\alpha(\vec{r}) = \langle\alpha\rangle + \delta\alpha(\vec{r}), \quad \beta(\vec{r}) = \langle\beta\rangle + \delta\beta(\vec{r}) \tag{8.31}$$



Thus, within this model flux quanta effectively move in a random potential. The respective minima represent the places, where the flux quanta are anchored. Physically, these places can be identified with the locations of lattice defects.

8.4.1 Typical Lattice Defects:

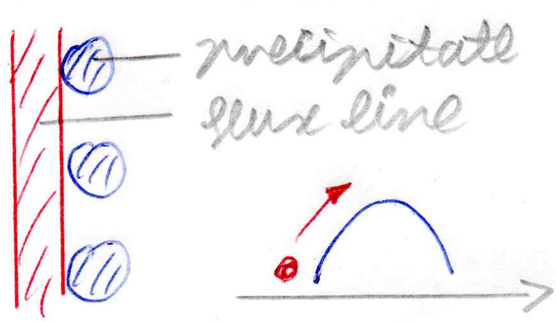
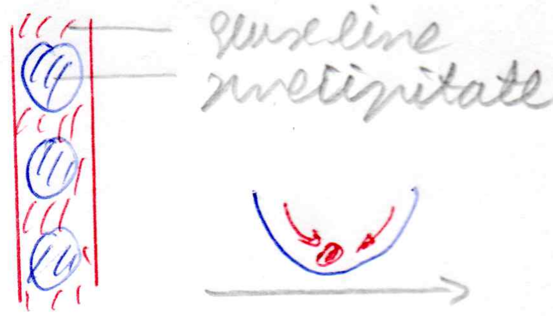
In principle one can distinguish 4 types of typical lattice defects depending on the involved dimensionality:

1) Point defects, which represent zero-dimensional lattice defects, are provided by vacancy defects ("Leerstellen") or interstitial defects ("Zwischengitteratome"), i.e. atoms are missing or occupy a site in the crystal structure at which there is usually not an atom. Such point defects generate an elastic long-range tension field $\sigma \sim 1/r^3$ but they do not affect the length scales λ_L and ξ of the Ginzburg-Landau theory.

2) Dislocations ("Versetzungen") are one-dimensional lattice defects, where only a half plane of atoms exists. They generate a longer ranged tension field $\sigma \sim 1/r$ but also not affect λ_L and ξ.

3) Grain boundaries ("Korngrenzen") occur, where the crystallographic direction of a lattice abruptly changes. This usually occurs where two crystals begin to grow separately and they meet. They represent two-dimensional lattice defects, which are basically tension-free. But as they rely on different atomic structures, they are described by different length scales λ_L and ξ.

4) Precipitates ("Ausscheidungen") represent three-dimensional lattice defects, where a different phase of matter occurs in a superconducting matrix. This changes λ_L and ξ as well as leads to an additional potential V. The presence of precipitates can either raise or lower the energy of the flux line.

$E > 0$	$E < 0$
The flux line is attached to the precipitates. Thus, a motion of the flux line costs energy.	The flux line sits on top of the precipitates. Thus, the presence of the precipitates releases energy.
	
condition: $H_c > H_c^{Matrix}$	condition: $H_c < H_c^{Matrix}$

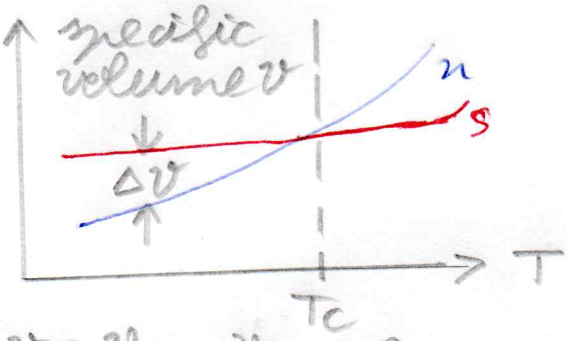
8.4.2 Examples:

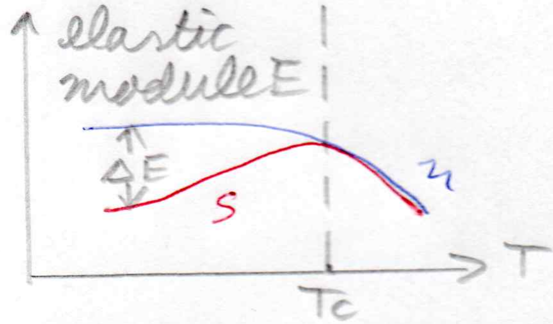
For technical applications one uses essentially only intermetallic superconductors:

1) Nb_3Sn: Point defects destroy superconducting properties.
2) NbTi: Point defects do not play a role. Due to a strong trapping of flux lines at precipitates one aims for achieving a larger upper critical field H_{c2}.

8.4.3 Origin of Tension Fields:

The generation of a flux quantum converts a superconducting region to a normal conducting one. The local change of the phase leads to a local change of mechanic and elastic quantities as, for instance, the specific volume, i.e. the volume per mass, or the elastic constants:





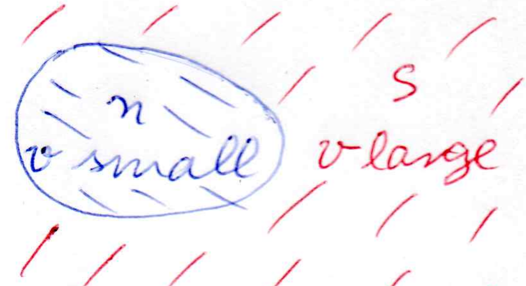
$\Delta v = v_n - v_s < 0$ $\qquad$ $\Delta E = E_n - E_s > 0$

Note that such changes of mechanic or elastic quantities have so far been neglected in the lecture.

8.4.4 Importance of Tension Fields:

The generation of a flux quantum implies a local phase transition from a super to a normal-conductor, which has according to the preceding subsection the following consequences:

1) The area of the flux quantum contracts and, thus, leads to a tensile stress ("Zugspannung"). This tension interacts with other lattice defects.



2) The elastic module increases, which leads to an increase of the elastic energy.

8.4.5 Comments:

1) A single flux quantum interacts with many defects. This necessitates a statistical treatment.
2) The interaction between the flux lines leads to the generation of a flux lattice. But such a flux lattice is locally deformed by single defects, which leads to a quite complicated spatial dependence of all involved physical quantities.
3) At the high Tc-superconductor $YBa_2Cu_3O_7$ oxigen defects occur, which have magnetic properties. Thus, there additional magnetic interaction energies have to be taken into account.

8.5 Nb_3Sn:

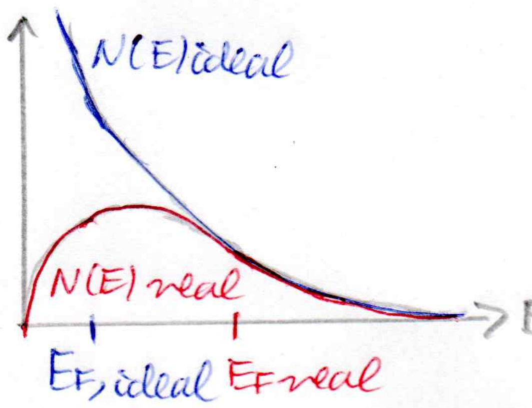
The material Nb_3Sn is technically quite interesting due to its critical temperature 18 K, which is among the highest of conventional superconductors. This intermetallic alloy is in particular important for high-field coils.

8.5.1 Critical Temperature:

The crystal structure of Nb_3Sn was already discussed in section 1.2. It is essential that the chains of Nb atoms do not touch each other, but have at least the distance of one half of the lattice constant. This justifies to treat Nb_3Sn approximately as a one-dimensional conductor. Thus, as already mentioned in section 1.4, this leads to a large density of states at the Fermi edge and to a large critical temperature (1.4) according to the BCS theory. Note that the same argument applies to Nb_3Ge, which even has the critical temperature 23 K.

8.5.2 Impact of Point Defects:

Point defects as vacancy or interstitial defects lead for Nb_3Sn to a reduction of length L of the one-dimensional potential box for electrons. As the energy eigenvalues are given by $E \sim 1/L^2$, this leads to an increase of the energies.

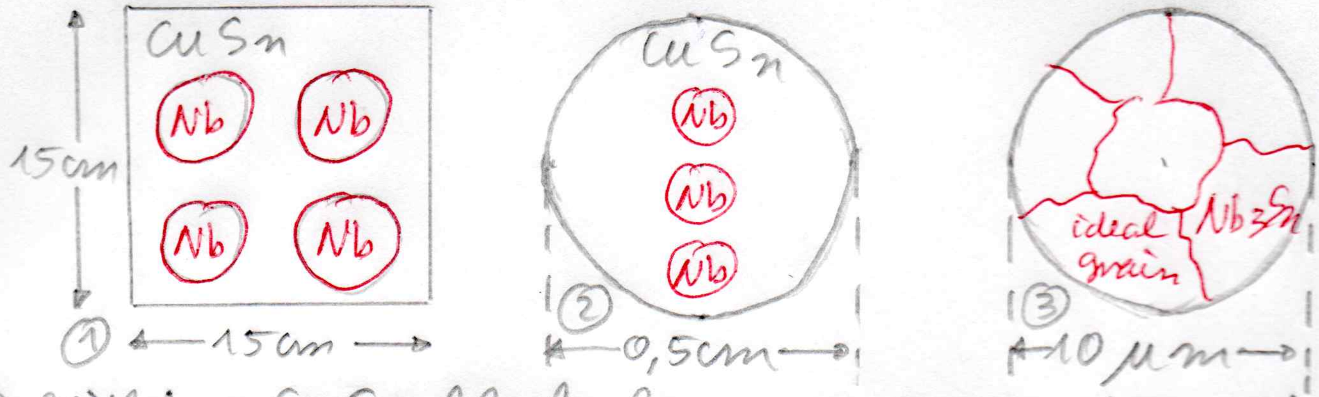


Thus, low-energetic states are eliminated and the density of states is reduced. And a lower density of states leads via (1.4) to a reduction of the critical temperature.

This leads to a dilemma for Nb_3Sn. On the one hand one needs point defects for the anchoring of flux lines in order to allow for high critical fields. But on the other hand those point defects reduce the critical temperature as just discussed.

8.5.3 Technical Solution:

In order to circumvent this dilemma one has designed a technical solution, which relies on the concrete process how Nb_3Sn is produced.



① Within a CuSn block of macroscopic size cylindrical holes are drilled, which are filled by Nb.

② Then the block is stretched such that small Nb bundels appear.

③ Glowing the arrangement at $T = 900°C$ initiates that Sn atoms slowly diffuse into the Nb and create tiny Nb_3Sn crystals. Within such a Nb_3Sn crystal grain boundaries emerge due to the tension of the material.

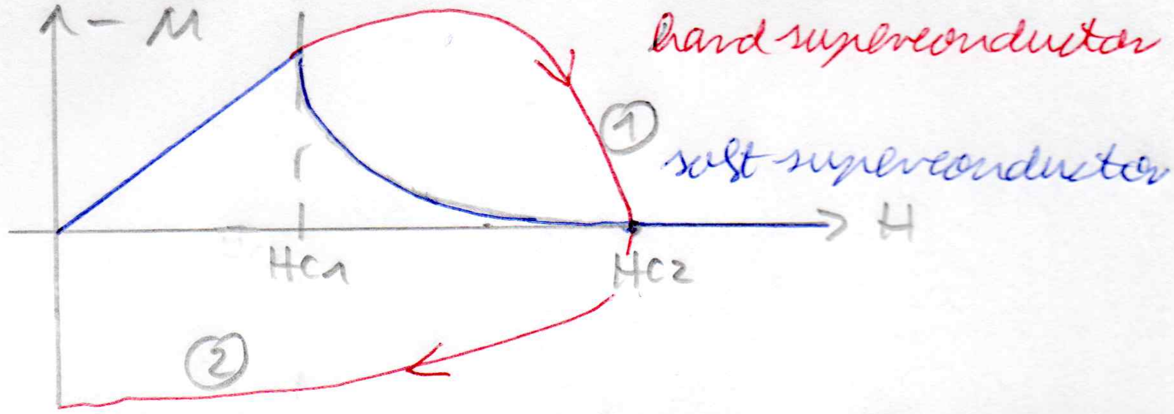
Within an ideal grain the critical temperature amounts to 18 K. At the same time the grain boundaries between the grains represent anchor centers for the flux lines, thus allowing also for large critical fields.

8.6 Hard Superconductors:

The above facts suggest to distinguish between soft and hard superconductors:

soft superconductor	hard superconductor
homogeneous material	inhomogeneous material
Flux lines do not energetically favour positions and can, thus, be freely moved around.	Flux lines are found preferably at the energetic minima of lattice defects and are, thus, not freely movable.
reversible magnetization curve	irreversible magnetization curve

The magnetization curve of a hard superconductor is highly irreversible. When the magnetic field is increased up to H_{c2}, the magnetization is negative, so we have diamagnetism, see ①. But when the magnetic field is again below H_{c2}, the magnetization is positive, so we have paramagnetism.



An example for such a hard superconductor is the sintered magnet YBaCuO. Sintering is a process, which is applied to ceramic materials. Due to heating a fine-grained material is transformed to a solid. As a result of the sintering of YBaCuO one yields a high Tc-superconductor with grain boundaries. Due to an additional radiation with an electron beam also point defects are created. With this one obtains a material with a much stronger irreversibility of the magnetization curve as for a usual type II superconductor. Therefore, such hard superconductors are sometimes also called type III superconductors.