TU KAISERSLAUTERN SS 2022
Department of Physics Priv.-Doz. Dr. Axel Pelster

## Quantum Field Theory — Problem Sheet 5

### Problem 11: Euler-Lagrange Equations

Assume that the action $\mathcal{A}$ of a field theory is given by the Lagrange density $\mathcal{L}$ via

$$\mathcal{A} = \frac{1}{c} \int_\Omega d^4x \, \mathcal{L} \left( \psi^\sigma(x^\lambda); \partial_\mu \psi^\sigma(x^\lambda) \right) . \tag{1}$$

**a)** The field-theoretic Hamilton principle states that the action is extremized with the help of the functional derivative

$$\frac{\delta \mathcal{A}}{\delta \psi^\sigma(x^\lambda)} = 0 . \tag{2}$$

Derive with this the Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \psi^\sigma(x^\lambda)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \left( \partial_\mu \psi^\sigma(x^\lambda) \right)} = 0 . \tag{3}$$

(4 points)

**b)** The Lagrange density of the Maxwell field in vacuum reads

$$\mathcal{L} = \alpha \, F_{\mu\nu} \, F^{\mu\nu} , \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{4}$$

Show that the corresponding Euler-Lagrange equations reduce to the Maxwell equations in the vacuum. (6 points)

### Problem 12: Symmetrization of Energy-Momentum Tensor

**a)** Determine the canonical momentum tensor $\Theta^{\mu\nu}(x^\lambda)$ for the Maxwell field and show that it is neither symmetric nor gauge invariant. (6 points)

**b)** Perform the Belifante construction for obtaining the symmetrized energy-momentum tensor $T^{\mu\nu}(x^\lambda)$ from the the canonical momentum tensor $\Theta^{\mu\nu}(x^\lambda)$ by the example of the Maxwell field. Determine the constant $\alpha$ such that energy density $u = cT^{00}$ reduces to the usual expression in SI units:

$$u = \frac{1}{2} \varepsilon_0 \mathbf{E}^2 + \frac{1}{2\mu_0} \mathbf{B}^2 . \tag{5}$$

Is the symmetric energy-momentum tensor of the Maxwell field gauge invariant? (8 points)

**Drop the solutions in the post box on the 5th floor of building 46 or, in case of illness/quarantine, send them via email to jkrauss@rhrk.uni-kl.de until May 31 at 14.00.**