Quantum Field Theory

= introduction to Quantum Electrodynamics (QED) ≙ one main building block of the standard model of elementary particle physics

1. Introduction:

1.1 Standard Model:

- describes 3 of 4 fundamental interactions
- basic concept: local gauge theory
  - free massive particles: global gauge invariance of phase
  - postulate: extension to local gauge invariance
  - compensation of extra terms (derivatives of phase) via gauge fields
  - derive constructively the interactions between massive particles and the gauge particles
- Overview:

interaction	gauge symmetry	gauge bosons
electromagnetic	$U(1)$	photon: γ
weak	$U(2)$	intermediate vector bosons $W^\pm$, Z^0
strong	$SU(3)$	gluons (8 versions)

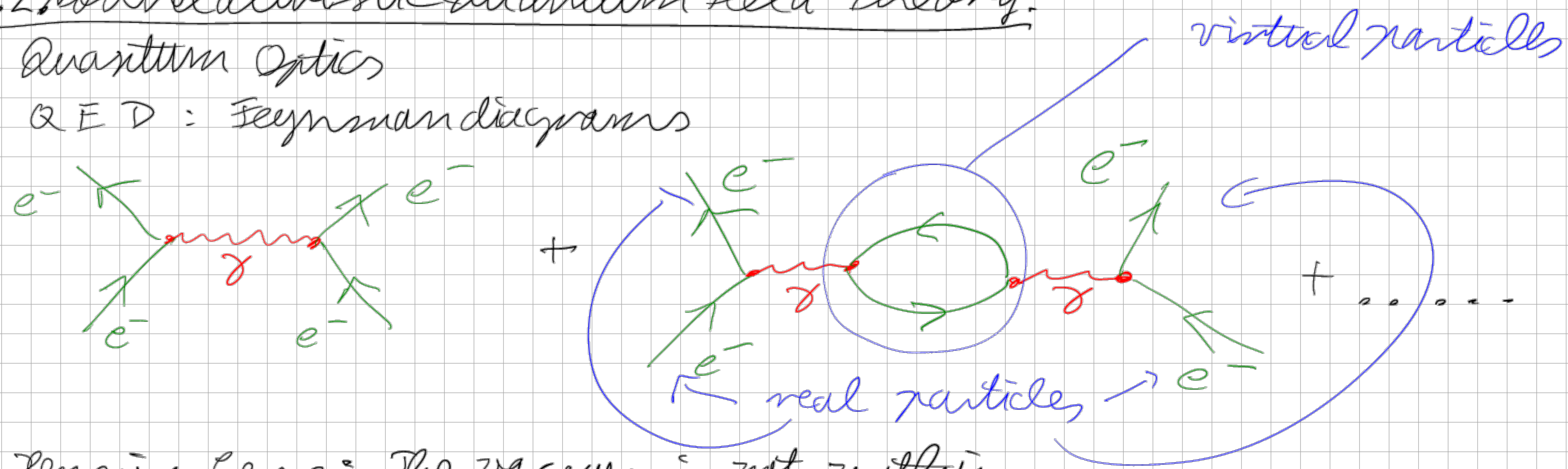
- QED is the theory in all natural sciences whose predictions agree most precisely with all experimentally available data

accuracy: $\hat{=}$ hair thickness resolved by

10 orders of magnitude R. F. look from east to west coast of US

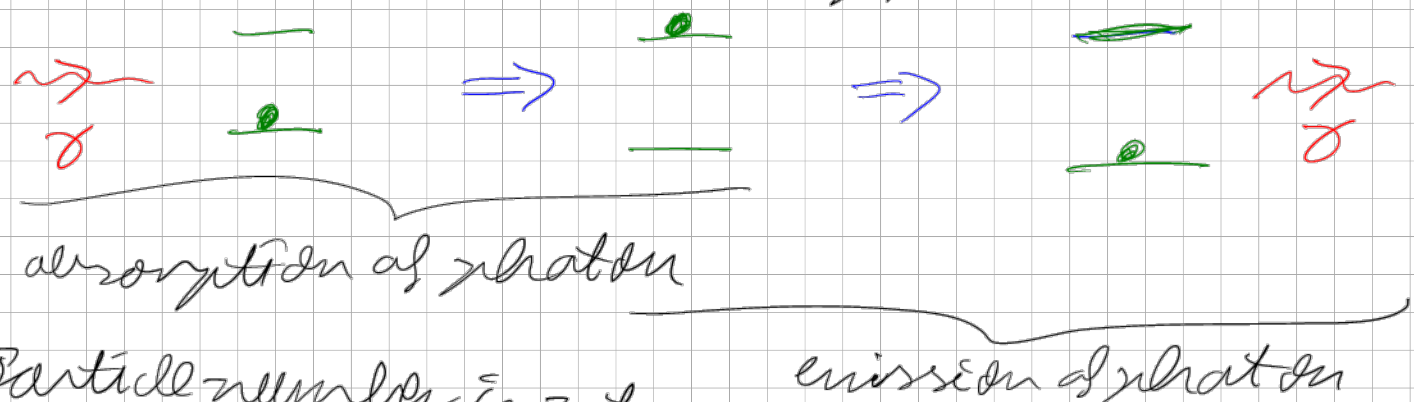
1.2 Nonrelativistic Quantum Field Theory:

- Quantum Optics
- QED: Feynman diagrams



Henning Genz: The vacuum is not nothing.

- Wanted: quantum mechanical formalism which is capable of describing an arbitrary number of particles
- "First quantization" is not appropriate:



- Particle number is not conserved for Einstein's elementary processes
- Only eigenstates and transition amplitudes are accessible in first quantization

- Identical particles: same properties as mass, spin, charge, ...
  - Classical world: distinguishable, i.e. can be enumerated
  - Quantum world: indistinguishable, i.e. can not be enumerated
  - But still artificial enumeration is necessary
  - Restriction: physical observables must be invariant with respect to any change of particle labelling → special symmetry requirement for any many body wave function
- Spin-statistics theorem of Wolfgang Pauli (1940)
  - unifying quantum mechanics and special relativity

- In 3 spatial dimensions only two kinds of indistinguishable particles exist

bosons	fermions
integer spin	half-integer spin
particles mediating forces	matter particles
Bose-Einstein statistics	Fermi-Dirac statistics
symmetric many-body wave functions	asymmetric many-body wave function

- (Anti-) symmetrization of many-body wave function quite cumbersome for large particle numbers → "second quantization"
  - introduction of creation/annihilation operators for particles
  - (Anti) symmetry of many-body wave function is automatically taken into account
  - derivable within "canonical field quantization"
  - Quantize "first quantized" Schrödinger theory: possible for both bosons and fermions
- Important applications in solid-state physics and is indispensable for QED
- Disclaimer: path/functional integral formalism to second quantization; not discussed in this module

1.3 Relativistic Fields and Their Quantization:

- Poincaré group: fundamental symmetry of space-time in absence of gravity
  - rotations, boosts, translations
  - apply concepts of Lie groups / algebras
- Casimir operators:
  - operators commuting with all generators of rotations, boosts, translations
  - all states in relativistic QFT are classified with respect to the Casimir operators of Poincaré group

spin	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	...
mass > 0	Higgs, $\pi^{\pm}$, π^0	electrons, quarks	$W^{\pm}$, Z^0	Δ resonances		...
mass = 0		neutrinos	photon, gluon		graviton	...

relativistic QFT = representation theory of Poincaré group
- Examples of free theory: Dirac and Maxwell theory
  - group-theoretical construction of free solutions
  - solve first Dirac eq. in rest frame and then boost in general frame
  - second quantize Dirac and Maxwell theory: propagators
- Noether theorem:
  - relation between symmetries and conservation laws
  - concrete example: QED

1.4 Quantum Electrodynamics:

Wednesday	8.00	
Friday	12.00	~~8~~ 4
Friday	14.00	4 ⇐