1.4 Quantum Electrodynamics:

- derive light-matter interaction via local gauge invariance ("minimal coupling")
- Use second quantization to perform perturbation theory with respect to light-matter interaction
  - Non-covariant Coulomb gauge for Maxwell field yields – finally – a covariant perturbative correction
  - Graphical representation of perturbative corrections in terms of Feynman diagrams
  - Goal: Construction of Feynman diagrams via graphical recursion relation ≙ cutting lines of Feynman diagrams of lower orders and gluing the fragments together with new vertices
- Cross-sections of scattering processes
  - Mott scattering (relativistic version of Rutherford scattering) $e^- Z \to e^- Z$
  - Møller scattering $e^- e^- \to e^- e^-$ ← our example
  - Bhabha " $e^- e^+ \to e^- e^+$
  - Compton scattering $e^- \gamma \to e^- \gamma$

$\Rightarrow$ lower orders: finite, higher orders: infinite concrete quantitative prediction for measurement of cross-section is not possible

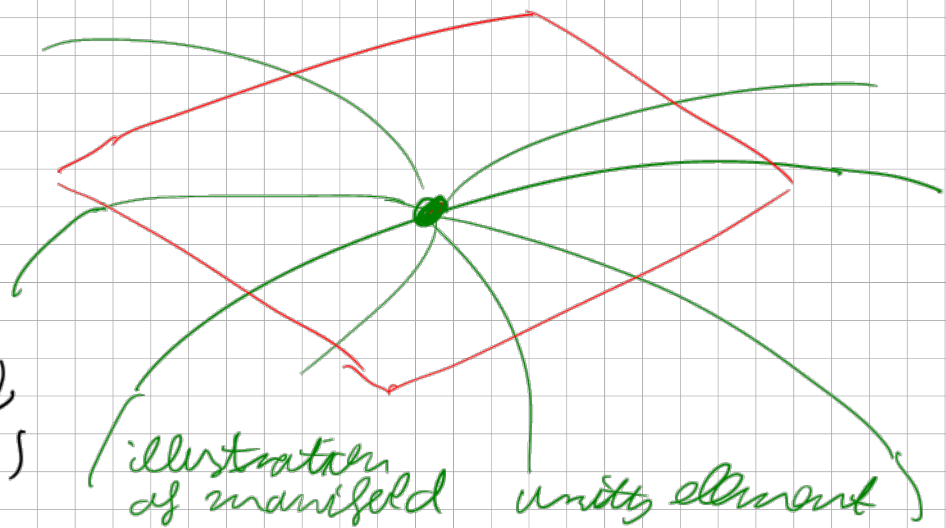
- Renormalization procedure:
  - regularise integrals: introduce additional calculational degree of freedom such that integrals become finite
  - examples:
    > Λ cut-off in momentum integrals: limit $\Lambda \to \infty$ recovers infinity
    > dimensional regularization: $D = 4 - \varepsilon$ (Gerard t'Hooft + Veltman)
      limit $\varepsilon \to 0$ yields infinities
    + many other regularization procedures
  - Infinities can be absorbed by the few parameters of the theory, i.e. mass, coupling constant and fields
  - Renormalization scheme in lowest order: ?
  - Renormalization to all orders: proven by Dyson

2 Poincaré Group:

Motivation:

- special relativity: space-time in absence of gravity (= Minkowskian structure)
- group symmetry, which leaves the Minkowskian structure invariant, is Poincaré group

- Lie group:
  - unifies mathematical structures of groups and manifold
  - group elements depends continuously and differentially on certain parameters
- Poincaré group: 10 parametric, non-abelian Lie group containing rotations (3 parameters), boosts (3 parameters), translations (4 parameters)



- Lie algebra: tangent plane
- Poincaré algebra: generators of rotations, boosts, translations
- Lie theorem: Lie group $\equiv \exp\{$"Lie algebra $\cdot$ parameters"$\}$

  defined by Taylor expansion

- Casimir operators:
  - commute with all elements of Lie algebra
  - eigenvalues characterise (irreducible) representations of Poincaré
  - each elementary particle belongs to one of those irreducible representations

2.1 Special Relativity:

- Formulated by Einstein in 1905: space-time in absence of gravity
- Based on two postulates of special relativity

  E1: The velocity of light is the same in all inertial systems

E2: The fundamental laws of physics have the same form in all inertial systems.

- concrete physical consequences: fast moving objects, checked on a daily basis at LHC (large hadron collider, CERN)
- Example: time dilation

for an observer in an inertial reference frame, a clock that is moving with constant velocity, goes slower than a clock in its frame of reference

- Special relativity unifies the description of space
- Contravariant space-time four-vector

$$(x^\mu) = (x^0, x^1, x^2, x^3) = (ct, x^i) = (ct, \vec{x})$$

latin index $i = 1, 2, 3$; greek index: $\mu = 0, 1, 2, 3$

- Light ray in two different inertial systems:

$$(ct)^2 - \vec{x}^2 = 0 = (ct')^2 - \vec{x}'^2$$

- Covariant Minkowski metric:

$$(g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x_\mu x^\mu = x'_\mu x'^\mu$$

invariance of scalar product: $g_{\mu\nu} x^\mu x^\nu = g_{\mu\nu} x'^\mu x'^\nu$

Einstein convention: summation over identical co/contravariant indices is implied

contraction

- Covariant space-time four vector: $x_\mu = g_{\mu\nu} x^\nu$

"pull down contravariant index"

with Minkowski metric to get covariant index

$$(x_\mu) = (x_0, x_1, x_2, x_3) = (ct, -x^i) = (ct, -\vec{x})$$

- Identity: $g_{\mu\nu}\,\delta^\nu{}_\kappa = g_{\mu\kappa}$, $\qquad (\delta^\nu{}_\kappa) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
- Contravariant Minkowski metric:

$$(g^{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \qquad g^{\mu\nu}\, g_{\nu\kappa} = \delta^\mu{}_\kappa \equiv g^\mu{}_\kappa$$

- Reconstruction: pull up indices

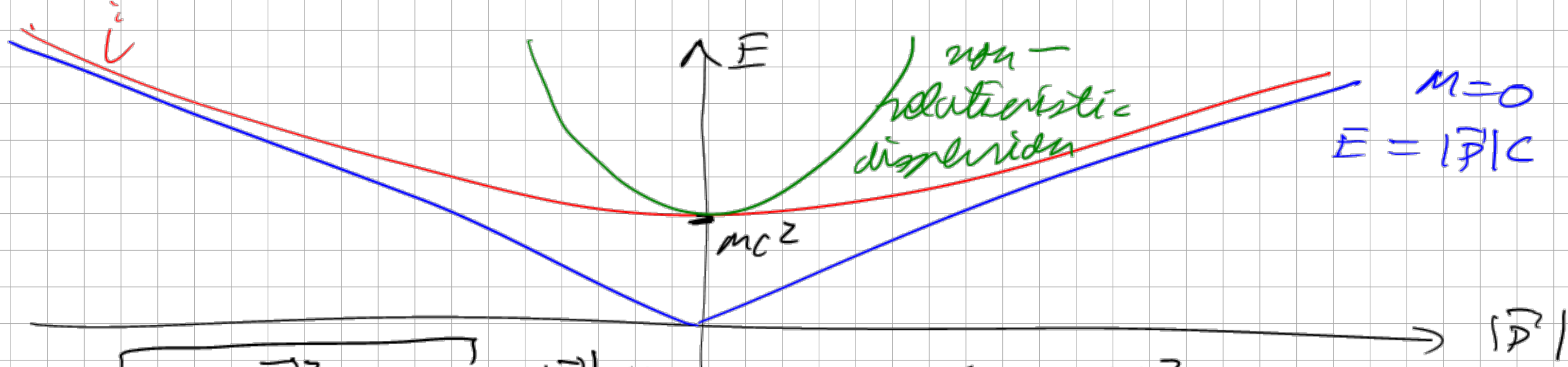
$$g^{\mu\nu} x_\nu = \underbrace{g^{\mu\nu}\, g_{\nu\kappa}}_{= g^\mu{}_\kappa = \delta^\mu{}_\kappa} x^\kappa = x^\mu$$

- concept of space-time four-vectors can be generalised to general four-vectors
- Another example: relativistic energy-momentum dispersion

$$E^2 = m^2c^4 + \vec{p}^{\,2}c^2 , \qquad E'^2 = \underbrace{m'^2}_{=m^2} c^4 + \vec{p}\,'^2 c^2$$

(rest mass identical in all inertial systems = Lorentz scalar)

$$\Rightarrow \left(\frac{E}{c}\right)^2 - \vec{p}^{\,2} = \left(\frac{E'}{c}\right)^2 - \vec{p}\,'^2 = m^2c^2$$



$$E = mc^2\sqrt{1+\frac{\vec{p}^{\,2}}{m^2c^2}} \;\xrightarrow{|\vec{p}| \ll mc}\; mc^2\left(1+\frac{\vec{p}^{\,2}}{2m^2c^2}+\dots\right) = mc^2 + \frac{\vec{p}^{\,2}}{2m} + \dots$$

- Contravariant four-momentum vector:

$$(p^\mu) = (p^0, p^1, p^2, p^3) = \left(\frac{E}{c}, p^i\right) = \left(\frac{E}{c}, \vec{p}\right)$$

$$\Rightarrow \; g_{\mu\nu}\, p^\mu p^\nu = g_{\mu\nu}\, p'^\mu p'^\nu$$

- Covariant four-momentum vector

$$(p_\mu) = (p_0, p_1, p_2, p_3) = \left(\frac{E}{c}, -p^i\right) = \left(\frac{E}{c}, -\vec{p}\right)$$

$$\Rightarrow \; \underbrace{p_\mu p^\mu}_{\text{scalar product}} = p'_\mu p'^\mu$$

- Scalar product of four-momentum vector is given by rest-mass

$$p^\mu p_\mu = m^2c^2$$