# Quantum Optics — Problem Sheet 11

**Problem 24: Atom Interferometer** **(10 points)**

In this exercise we derive a formalism for atomic interferometry, which is based on the optical Bloch equations with vanishing detuning $\Delta$.

**a)** Solve the optical Bloch equations for the Bloch vector $\mathbf{s}(t)$ in the special case $\Delta = 0$, i.e.

$$\dot{\mathbf{s}}(t) = -\Omega_{\mathrm{R}}\mathbf{e}_1 \times \mathbf{s}(t)\,, \tag{1}$$

where $\Omega_{\mathrm{R}}$ denotes the Rabi frequency. Write down an explicit expression for the time-evolution operator $\hat{U}(t)$.

**b)** Consider now as the initial state that the atom is in the ground state. What is the final state for times $\Omega_R t = \pi/2$ and $\Omega_R t = \pi$? How does the operator for measurement of the population imbalance $s_3$ read, and how can you calculate the actual population of ground and excited state from this operator?

**c)** Can you construct a beam splitter for atoms with your results from **a)** and **b)**?

**d)** Describe how you can build up an atomic Mach-Zehnder interferometer and calculate the evolution of the atom beam through the interferometer.

**e)** Explain qualitatively how the results change in presence of a detuning $\Delta$.

**Problem 25: Bloch Equations and Decay** **(14 points)**

Real experiments with atomic beams suffer from several decay mechanisms like population decay via spontaneous emission, which is modelled via the $T_1$-time, and decoherence of the atomic phases through varying detuning $\Delta$, which is described via the $T_2$-time. In this exercise, we will simulate these processes stochastically.

**a)** Implement the optical Bloch equations with detuning $\Delta$

$$\dot{\mathbf{s}}(t) = -\begin{pmatrix} \Omega_R \\ 0 \\ \Delta \end{pmatrix} \times \mathbf{s}(t) \tag{2}$$

numerically.

In the following we add peu à peu the stochastic processes.

**b)** At first we focus on the spontaneous decay, which means that at some random time the excited states decay into the ground state.
The spontaneous emission is included numerically by drawing a uniformly distributed real number $r$ from the interval $[0, 1]$. If $r < \Delta t\gamma$, where $\Delta t$ is the time step and $\gamma$ the decay rate, we set $s_3 = -1$, otherwise we perform a normal time step. Show numerically that in the special case $\Omega_R = 0$ this procedure yields in the average of many realisations

an exponential damping $e^{-\gamma t}$ for the excited state population. Which initial condition makes the most sense? What happens for finite frequency $\Omega_R$?

**c)** We come now to the decoherence, which means that the detuning $\Delta$ is not constant but varies from atom to atom. We assume here a Gaussian distributed detuning, which can arise, for instance, in atomic beams through the Doppler effect.
Use your implementation from **a)** with the initial state $\mathbf{s}_0 = (0, 1, 0)^T$ and a Gaussian distributed detuning $\Delta$ with $\langle\Delta\rangle = 0$ and $\langle\Delta^2\rangle \sim T$, where $T$ is the temperature of the surrounding introducing the Doppler shift. Show that this leads in the ensemble average to a decay of the Bloch vector.

**d)** We now combine the effects from **b)** and **c)** and implement a typical experimental sequence, namely the spin-echo sequence. At the beginning, the atom is considered to be in the ground state. The sequence consists of a $\pi/2$-pulse followed by a waiting time $\tau$, then a $\pi$-pulse again followed by a waiting time $\tau$ and finally a $\pi/2$-pulse again. What do you observe for the Bloch vector for different waiting times $\tau$ and how do you explain your findings?

**e)** Implement your Mach-Zehnder interferometer from **d)** and add spontaneous decay and decoherence. How do the results change?

**Drop the solutions in the post box on the 5th floor of building 46 or, in case of illness/quarantine, send them via email to jkrauss@rhrk.uni-kl.de until July 7 at 12.00.**