RPTU Kaiserslautern-Landau
Department of Physics

Summer Term 2023

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Quantum Optics — Problem Sheet 12

Problem 26: Detection of Excited Atoms (10 points)

In this exercise we calculate the probability of measuring whether a given atom is at the beginning of the experiment in the excited state. For this purpose we consider two atoms with energy levels E_{gi} and E_{ei} with $i = 1, 2$, interacting according to the Jaynes-Cummings model with a single light mode at frequency ω. The states of the system are denoted by $|\downarrow\downarrow n\rangle$, $|\uparrow\uparrow n\rangle = \sigma_{1+}\sigma_{2+}\frac{\hat{a}^{\dagger n}}{\sqrt{n!}}|\downarrow\downarrow 0\rangle$, as well as correspondingly $|\uparrow\downarrow n\rangle$ and $|\downarrow\uparrow n\rangle$.

a) Write down the Hamiltonian for this problem by assuming the rotating-wave approximation. The coupling strengths of the two atoms to the light field are assumed to be equal.

b) Transform the Hamiltonian and the corresponding operators from **a)** into the interaction picture. How does the time evolution operator $\hat{U}(t)$ formally read in the interaction picture? How can you expand this formal expression in orders of the coupling strength?

c) Calculate up to the first non-vanishing order in the coupling strength the transition amplitude $\langle\uparrow\downarrow 0|\hat{U}(t)|\downarrow\uparrow 0\rangle$. Can you explain the arising order physically?

d) Plot the resulting time-dependent transition probability.

e) Calculate the probability that the system stays in the initial state, i.e. $|\langle\downarrow\uparrow 0|\hat{U}(t)|\downarrow\uparrow 0\rangle|^2$ up to the same order in the coupling strength.

Problem 27: Numerics of Jaynes-Cummings Model (14 points)

In this problem we aim for a numerical implementation of the Jaynes-Cummings model. As a basis we choose $|\downarrow\rangle = (0, 1)^T$ and $|\uparrow\rangle = (1, 0)^T$ for the atom and $|n\rangle = (\delta_{n+1,m})_m^T$, where we count from below, and we identify the photon vacuum with $|0\rangle = (\ldots, 0, 0, 1)^T$. **Note:** You can ignore the $+1$ in the photon vector, if your programming language starts array indices at 0.

a) Write down and implement the needed operators in the above basis, i.e. the operators $\sigma_z \otimes \mathrm{Id}$ with $\sigma_z = \sigma_+\sigma_- - 1/2$, $\sigma_\pm \otimes \mathrm{Id}$ for the atom and $\mathrm{Id} \otimes \hat{a}$, $\mathrm{Id} \otimes \hat{a}^\dagger$ for the photon mode and the operators $\hat{a} \otimes \sigma_+$, $\hat{a}^\dagger \otimes \sigma_-$ for the interaction Hamiltonian, where $\otimes$ denotes the Kronecker product and Id the corresponding identity operator. The vector of the system state is in this case given by $|\eta, n\rangle = |\eta\rangle \otimes |n\rangle$, where $\eta = g, e$, e.g. the ground state of the full system is given by $|g, 0\rangle = (0, 1)^T \otimes (\ldots, 0, 0, 1)^T$.
Hint: If you are not familiar with the Kronecker product, just look it up on Wikipedia.

Attention: Take always care of a reasonable truncation of your photon state vector for each simulation. For **b)** and **c)** we only allow one photon at maximum.

b) Implement the Jaynes-Cummings Hamiltonian with your matrices from **a)** and construct a solver for the corresponding time evolution.

c) Consider now as an initial state the atom to be in the excited state and the photon field in the vacuum state. Measure the time evolution of the operator σ_z and the photon number operator $\hat{n} = \hat{a}^\dagger \hat{a}$. Interpret your results physically.

d) Take again as an initial state the atom to be in the excited state, but vary now the initial photon number from $N = 0$ to $N = 10$. Measure the same quantities as in **c)**. What do you note about the Rabi frequency? Can you explain the result?

e) Implement the displacement operator $\hat{D}(\alpha)$ and verify, that you can create coherent states.

f) Consider now as an initial state the atom in the ground state and the field in a coherent state $|\alpha\rangle$ with $\langle\alpha|\hat{n}|\alpha\rangle = 25$. What do you note? Take care of your photon state truncation!

Drop the solutions in the post box on the 5th floor of building 46 or, in case of illness/quarantine, send them via email to jkrauss@rhrk.uni-kl.de until July 14 at 12.00.