RPTU Kaiserslautern-Landau
Department of Physics

Summer Term 2023
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Quantum Optics

Problem Sheet 4

Problem 8: Coherence (12 points)

In this problem we work on the coherence properties of electrical fields in Fock states. Consider a one-dimensional electrical field with only one polarisation component in a plain cavity. Its operator we take in the form $\hat{E}(x,t) = \hat{E}^{(-)} + \hat{E}^{(+)}$, with $\hat{E}^{(+)} = \hat{E}^{(-)\dagger}$ and

$$\hat{E}^{(-)} = i \sum_{l=1}^{\infty} \sqrt{\frac{\hbar\omega_l}{2L\epsilon_0}} \sin(k_l x) e^{i\omega_l t} \hat{a}_l^\dagger \,. \tag{1}$$

a) The first-order coherence of the electric field is described by

$$G^{(1)}(x_1,t_1;x_2,t_2) = \left\langle \hat{E}^{(-)}(x_1,t_1)\hat{E}^{(+)}(x_2,t_2) \right\rangle . \tag{2}$$

How is this quantity related to a simple coherence measurement? Calculate (2) for the Fock state $|\{n_l\}\rangle$, with $n_l = n\delta_{ml}$.

b) Also of physical interest is the so-called contrast, which is the normalised correlation function

$$g^{(1)}(x_1,t_1;x_2,t_2) = \frac{G^{(1)}(x_1,t_1;x_2,t_2)}{\sqrt{G^{(1)}(x_1,t_1;x_1,t_1)G^{(1)}(x_2,t_2;x_2,t_2)}} . \tag{3}$$

Show that $g^{(1)}$ is bounded according to $0 \leq |g^{(1)}(x_1,t_1;x_2,t_2)| \leq 1$. Calculate $g^{(1)}$ for the situation in **a)**.

c) The normalised second-order degree of coherence is defined by the function

$$g^{(2)}(x_1,t_1;x_2,t_2) = \frac{\left\langle \hat{E}^{(-)}(x_1,t_1)\hat{E}^{(-)}(x_2,t_2)\hat{E}^{(+)}(x_2,t_2)\hat{E}^{(+)}(x_1,t_1) \right\rangle}{G^{(1)}(x_1,t_1;x_1,t_1)G^{(1)}(x_2,t_2;x_2,t_2)} . \tag{4}$$

Which kind of measurement does it describe? What is its value for the Fock state from **a)**? How is this result related to classical fields?

Problem 9: Hong-Ou-Mandel Effect (12 points)

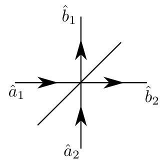

FIG. 1: Beam splitter with incoming photon $\hat{a}_1$, $\hat{a}_2$ and outgoing photons $\hat{b}_1$, $\hat{b}_2$.

In this problem we work out the Hong-Ou-Mandel effect, which is a purely quantum mechanical effect without classical analogue. Consider a 50/50 beam splitter with two incoming photons and two channels for the output, see Fig. 1.

a) Argue (with formulas!) that the beam splitter is described by the equation

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} . \tag{5}$$

b) Consider now the state $|\psi\rangle = \hat{a}_1^\dagger \hat{a}_2^\dagger |0\rangle$. Before doing the calculation, which outcome states do you suspect? Use now (5) to calculate the outcome states. What do you note?

c) What is the expectation value of having one photon in each output state? Can we measure one photon in each output state at the same time?

Drop the solutions in the post box on the 5th floor of building 46 or, in case of illness/quarantine, send them via email to jkrauss@rhrk.uni-kl.de until May 12 at 12.00.