RPTU Kaiserslautern-Landau | Summer Term 2023
Department of Physics | Priv.-Doz. Dr. Axel Pelster

Quantum Optics — Problem Sheet 8

Problem 17: Mach-Zehnder Interferometer (8 points)

In this exercise, we combine two beam splitters to a Mach-Zehnder interferometer like in Fig. 1. The box ϕ means, that a phase shift of ϕ is applied in the corresponding arm of the interferometer. We assume to have 50/50 beam splitters.

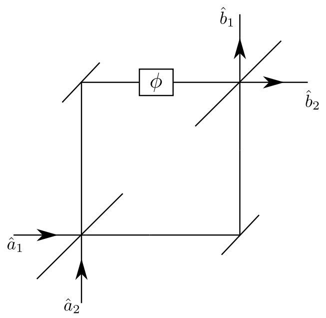

FIG. 1: Mach-Zehnder interferometer with incoming photons $\hat{a}_1$, $\hat{a}_2$ and outgoing photons $\hat{b}_1$, $\hat{b}_2$.

a) Like in the case of a single beam splitter derive the operator $\hat{U}(\phi)$ for the Mach-Zehnder interferometer, such that

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \hat{U}(\phi) \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}. \tag{1}$$

b) Calculate the output state for a single photon input state $\hat{a}_1^\dagger |0\rangle$. Is the output state any special state? Now, measure the ϕ-dependent photon number in both arms. What do you note? How do you explain the result?

c) Now send in two photons $\hat{a}_1^\dagger \hat{a}_2^\dagger |0\rangle$ and measure the photon number in both output channels and the coincidence of measuring one photon in each arm at the same time. Does the Hong-Ou-Mandel effect "survive"?

Problem 18: Stochastic ODE for Damped Harmonic Oscillator (16 points)

In this exercise we consider a stochastic ordinary differential equation (ODE) describing a damped harmonic oscillator with damping rate κ and oscillation frequency ω:

$$\dot{\alpha}(t) = -(\kappa + i\omega)\alpha(t) + \sqrt{2\kappa\bar{n}}\,\xi(t), \tag{2}$$

with the initial condition $\alpha(0) = 0$. Here, $\xi(t)$ denotes a complex stochastic variable with the expectation values $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi^*(t') \rangle = \delta(t - t')$. In the following you can treat $\xi(t)$ as an external source.

a) Solve the stochastic ODE (2) and calculate both the expectation value $\langle \alpha(t) \rangle$ and the correlation $\langle \alpha^*(t')\alpha(t) \rangle$ for $t' > t$. Specify now the latter to $t' = t$ in order to measure the average particle number $\langle n(t) \rangle = \langle a^*(t)a(t) \rangle$. As you note, this yields the same results as **Problem 15**. This is due to the fact, that the partial differential equation there and the stochastic ODE (2) here are equivalent descriptions of the same stochastic process.

b) Implement a solver of your choice for the deterministic part of the ODE (2), i.e. for the ODE

$$\dot{\alpha}(t) = -(\kappa + i\omega)\alpha(t)\,. \tag{3}$$

Verify that it works by choosing the initial condition $\alpha(0) = 1$.
Note: You do not need a high-order sophisticated solver here.

c) Now we add the stochastic noise simply by adding a normal-distributed random number ξ which is centered at 0 and has the standard deviation $\sqrt{\Delta t}$, where Δt is the length of a single time step. The random number ξ is redrawn in every single step. Compare your results to the analytical solution from **a)**.
Note: Depending on the programming language you can either draw directly a complex number, or you have to take two normal-distributed real numbers and add them up.
Bonus exercise (2 extra points): Explain, why the nomal distribution of ξ is centered at 0 and has the standard deviation $\sqrt{\Delta t}$.
Note: Please hand in your code by uploading it on seafile in your shared folder and do not hand it in by printing the code. Furthermore, please use common programming languages like Python, MatLab, etc. and only submit a running code. If your code is not running, it will not be corrected. Please take into account that you need some device like a computer etc. in order to present your solution to your colleagues in the exercises.

Drop the solutions in the post box on the 5th floor of building 46 or, in case of illness/quarantine, send them via email to jkrauss@rhrk.uni-kl.de until June 9 at 12.00.