QFT = quantum field theory

1. Introduction

this lecture = quantum electrodynamics (QED)

1.1 Standard Model:

- describes 3 of the 4 fundamental interactions
- basic concept: local gauge theory
  - free massive particles: global gauge invariance of phase
  - postulate: extension to local gauge invariance (≙ phase changing from space-time point to space-time point)
  - extra terms compensated by additional gauge field
  - derive interaction between the massive particle
- Overview:

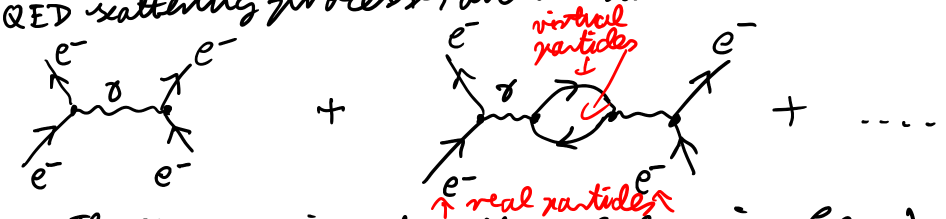
interaction	gauge symmetry	gauge bosons
electrodynamic	U(1)	photon
weak	U(2)	intermediate vector bosons
strong	SU(3)	gluons

- QED is the most precise theory in all natural sciences: comparison between theoretical predictions and experiments

accuracy: 10 orders of magnitude $\xrightarrow[\text{Richard Feynman}]{\text{comparison by}}$ hair thickness negligible by looking from West to East coast of the US

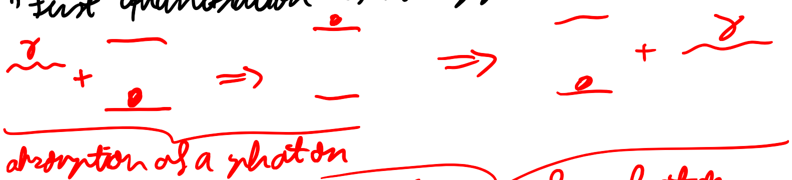
1.2 Non-Relativistic Quantum Many-Body:

- QED scattering processes are described by Feynman diagrams:



$\Rightarrow$ The vacuum is not nothing (Henning Gena)

- Wanted: quantum mechanical formalism which is capable of describing an arbitrary number of particles
- "First quantization" is not appropriate for that: elementary process of light-matter interaction ("Einstein processes") not covered by "first quantization", where number of particles is conserved



"first quantization" only deals with calculating energy eigenvalues or transition probabilities

- Identical particles = same properties as e.g. mass, spin, charge
  - Classical world: distinguishable, i.e. particles can be enumerated
  - Quantum world: indistinguishable, i.e. " "not" "
  - But still artificial enumeration of particles is necessary
  - Restriction: physical observables must be invariant with respect to any change of particle labelling $\Rightarrow$ special symmetry requirement for many-body wave functions
- Spin-statistic theorem of Wolfgang Pauli (1940):
  - Unifying special relativity and quantum mechanics
  - In 3 spatial dimensions only 2 kinds of indistinguishable particles exist: bosons and fermions
- Overview:

bosons	fermions
integer spin	half-integer spin
particles mediating forces	matter particles
Bose-Einstein statistics	Fermi-Dirac statistics
symmetric many-body wave function	anti-symmetric many-body wave function

- (Anti-)Symmetrization of many-body wave function quite cumbersome for large particle numbers ⇒ "Second quantization"
  - introduction of creation and annihilation operators for particles
  - (anti-) symmetry of many-body wave functions automatically taken into account
  - derivable within "Canonical Field Quantization"
  - Quantize "first quantized" Schrödinger theory both for bosons and fermions
- Important applications in solid-state physics and is indispensible for formulating QED
- Disclaimer: path/functional integral formulation of second quantization is not dealt with in this lecture

1.3 Relativistic Fields and Their Quantization:

- Poincaré group: fundamental space-time symmetry in the absence of gravity
  - rotations, boosts, translations
  - applying concepts of Lie groups / algebras
- Casimir operators: operators commuting with all generators of rotations, boost, translations
  - states in relativistic QFT are classified with respect to the eigenvalues of the Casimir operators of the Poincaré group
  - spin + mass

spin	0	1/2	1	3/2	2	- - - -
mass > 0	pions Higgs	leptons, quarks	intermediate vector bosons ($W^\pm, Z^0$)	Δ-resonances		
mass = 0		~~neutrinos~~	photon, gluon		graviton	

relativistic QFT = representation theory of Poincaré group

- Examples of free theories: Maxwell (and Dirac) theory
  - group-theoretical construction of free solutions with polarization (helicity) degrees of freedom
  - solve first Maxwell (Dirac) equations in particular reference frame (rest frame) and then rotate (boost) the solution to an arbitrary reference frame (inertial frame)
  - second quantize Maxwell (Dirac) theory and determine free propagators
- Noether theorem: relation between symmetries and conservation laws

1.4 Quantum Electrodynamics:

- derive light-matter interaction via local gauge invariance ("minimal coupling")
- use second quantization to perform perturbation theory with respect to the light-matter interaction strength
  - non-covariant Coulomb gauge for Maxwell field yields covariant perturbative corrections
  - graphical representation of perturbative corrections via Feynman diagrams
  - construct Feynman diagrams via graphical reasoning
- Cross-sections of scattering processes:

Mott scattering (relativistic Rutherford scattering)	$e^- Z - e^- Z$
Møller scattering	$e^- - e^-$
Bhabha "	$e^- - e^+$
Compton "	$e^- \gamma - e^- \gamma$

  - lowest order: finite
  - higher orders: infinities

  → concrete quantitative predictions no longer possible?
- Renormalization procedure:
  - regularize the integrals: introduce new degree of freedom such that integrals become finite
    - > Λ-cut offs in momentum integrals: limit $\Lambda \to \infty$ you recover infinities
    - > dimensional regularization: $D = 4 - \varepsilon$: limit $\varepsilon \to 0$ " " "
      (Gerard 't Hooft)
  - infinities are absorbed by parameters of theory: mass, coupling constant + fields
  - renormalization to all orders was proven by Freeman Dyson