5.5 Canonical Field Quantization:

Grassmann fields $\psi(\vec{x},t)$, $\pi(\vec{x},t)$ $\xrightarrow{\text{second quantization}}$ operators $\hat{\psi}(\vec{x},t)$, $\hat{\pi}(\vec{x},t)$

$$\{F, G\}_+ \longrightarrow \begin{cases} \frac{1}{i\hbar}[\hat{F},\hat{G}]_- & \text{(Poiss. brack. anti-symm.)} \\ \frac{1}{i\hbar}[\hat{F},\hat{G}]_+ & \text{(Poiss. brack. symmetric)} \end{cases}$$

depends on parity of F, G

1. Application: fundamental equal-time Poisson brackets
$\rightarrow$ equal-time anti-commutator relations

$$[\hat{\psi}(\vec{x},t), \hat{\psi}(\vec{x}',t)]_+ = 0 = [\hat{\pi}(\vec{x},t), \hat{\pi}(\vec{x}',t)]_+, \quad [\hat{\psi}(\vec{x},t), \hat{\pi}(\vec{x}',t)]_+ = - \delta(\vec{x}-\vec{x}')$$

$$\pi(\vec{x},t) = - i\hbar\, \psi^*(\vec{x},t) \longrightarrow \hat{\pi}(\vec{x},t) = - i\hbar\, \hat{\psi}^+(\vec{x},t)$$

$$[\hat{\psi}^+(\vec{x},t), \hat{\psi}^+(\vec{x}',t)]_+ = 0, \quad [\hat{\psi}(\vec{x},t), \hat{\psi}^+(\vec{x}',t)]_+ = \delta(\vec{x}-\vec{x}')$$

2. Application: Hamilton equations $\rightarrow$ Heisenberg equations

$$\frac{\partial\psi(\vec{x},t)}{\partial t} = \{\psi(\vec{x},t), H\}_+ \qquad i\hbar\frac{\partial\hat{\psi}(\vec{x},t)}{\partial t} = \cancel{\frac{1}{i\hbar}}[\hat{\psi}(\vec{x},t), \hat{H}]_-$$

$$\frac{\partial\pi(\vec{x},t)}{\partial t} = \{\pi(\vec{x},t), H\}_+ \longrightarrow i\hbar\frac{\partial\hat{\pi}(\vec{x},t)}{\partial t} = \cancel{\frac{1}{i\hbar}}[\hat{\pi}(\vec{x},t), \hat{H}]_-$$

$$\hookrightarrow i\hbar\frac{\partial\hat{\psi}^+(\vec{x},t)}{\partial t} = [\hat{\psi}^+(\vec{x},t), \hat{H}]_-$$