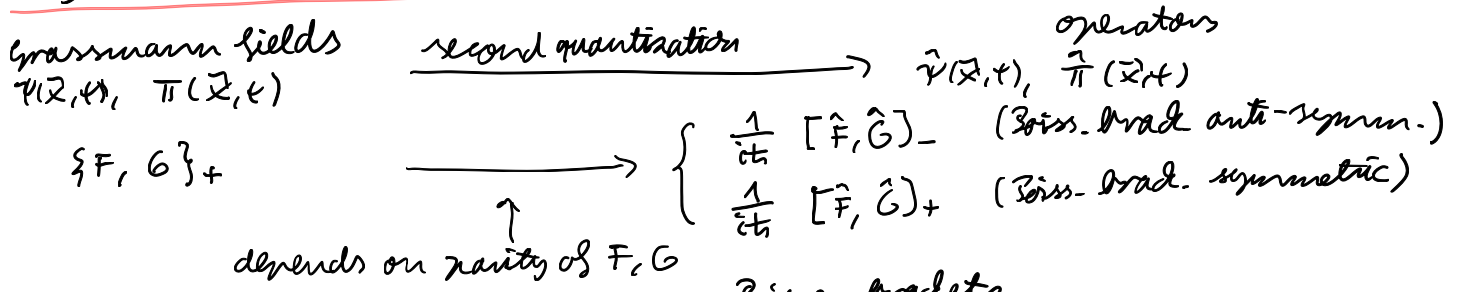


5.5 Canonical Field Quantization:



1. Application: fundamental equal-time Poisson brackets

\rightarrow equal-time anti-commutator relations

$$[\hat{\psi}(\vec{x}, t), \hat{\psi}(\vec{x}', t)]_+ = 0 = [\hat{\pi}(\vec{x}, t), \hat{\pi}(\vec{x}', t)]_+, \quad [\hat{\psi}(\vec{x}, t), \hat{\pi}(\vec{x}', t)]_+ = -\delta(\vec{x} - \vec{x}')$$

$$\pi(\vec{x}, t) = -i\hbar \psi^*(\vec{x}, t) \quad \longrightarrow \quad \hat{\pi}(\vec{x}, t) = -i\hbar \hat{\psi}^\dagger(\vec{x}, t)$$

$$[\hat{\psi}^\dagger(\vec{x}, t), \hat{\psi}^\dagger(\vec{x}', t)]_+ = 0, \quad [\hat{\psi}(\vec{x}, t), \hat{\psi}^\dagger(\vec{x}', t)]_+ = \delta(\vec{x} - \vec{x}')$$

2. Application: Hamilton equations \rightarrow Heisenberg equations

$$\frac{\partial \psi(\vec{x}, t)}{\partial t} = \{ \psi(\vec{x}, t), H \}_+ \quad \longrightarrow \quad i\hbar \frac{\partial \hat{\psi}(\vec{x}, t)}{\partial t} = \frac{1}{i\hbar} [\hat{\psi}(\vec{x}, t), \hat{H}]_-$$

$$\frac{\partial \pi(\vec{x}, t)}{\partial t} = \{ \pi(\vec{x}, t), H \}_+ \quad \longrightarrow \quad i\hbar \frac{\partial \hat{\pi}(\vec{x}, t)}{\partial t} = \frac{1}{i\hbar} [\hat{\pi}(\vec{x}, t), \hat{H}]_-$$

$$\hookrightarrow i\hbar \frac{\partial \hat{\psi}^\dagger(\vec{x}, t)}{\partial t} = [\hat{\psi}^\dagger(\vec{x}, t), \hat{H}]_-$$